Vassiliev Invariants
(Chord parametrization)
Part I

Evert Stenlund

$[\![2]\!] =$ $[\![3]\!] =$

$[\![41]\!] =$ $[\![42]\!] =$ $[\![43]\!] =$

$[\![51]\!] =$ $[\![52]\!] =$ $[\![53]\!] =$

$[\![54]\!] =$

$[\![61]\!] =$ $[\![62]\!] =$ $[\![63]\!] =$

$[\![64]\!] =$ $[\![65]\!] =$ $[\![66]\!] =$

$[\![67]\!] =$ $[\![68]\!] =$ $[\![69]\!] =$
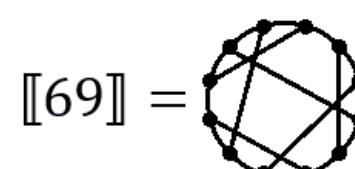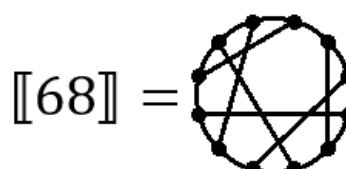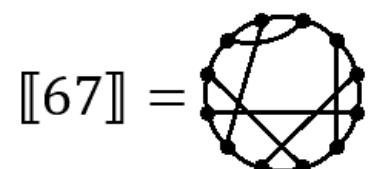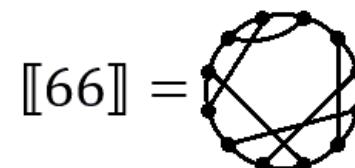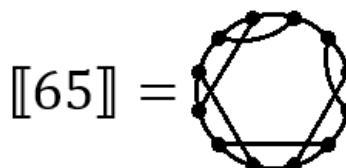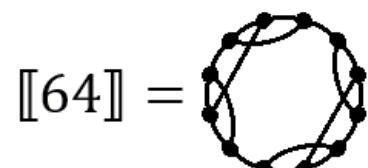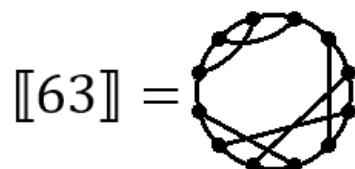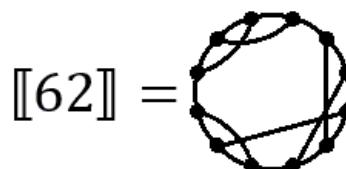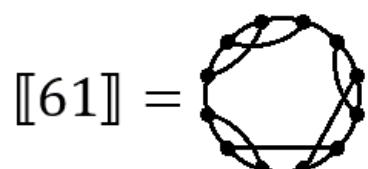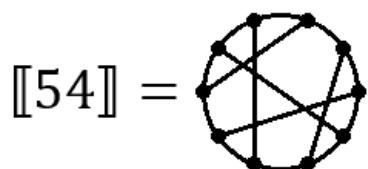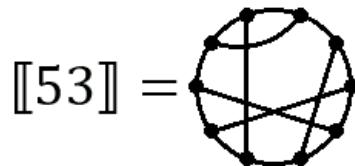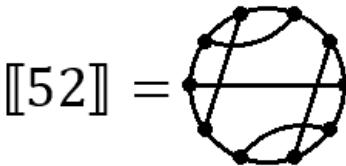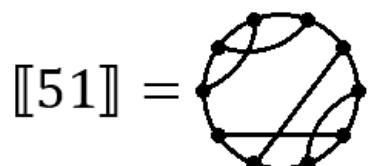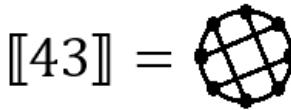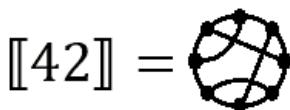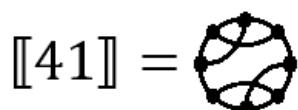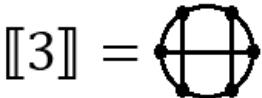
3_1:

$v_{even} = x_{3.1}$

$v_{odd} = \pm y_{3.1}$
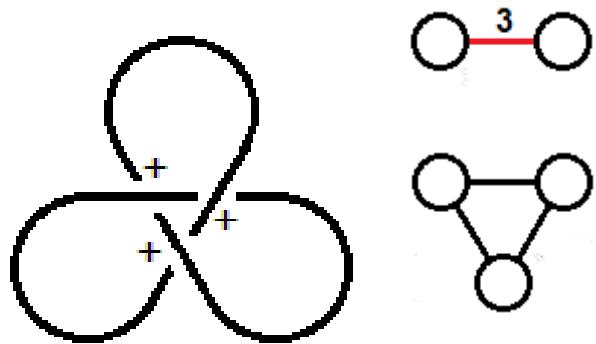
v_2	$[\![2]\!]$
v_3	$\pm[\![3]\!]$
v_4	$[\![2]\!]$
v_5	$\pm[\![3]\!]$
v_6	$[\![2]\!]$

4_1:

$$v_{even} = x_{4.1}$$

$$v_{odd} = 0$$
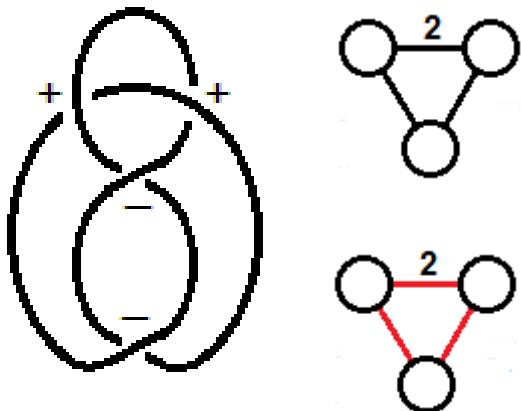
v_2	$-[\![2]\!]$
v_3	0
v_4	$\dfrac{1}{2}([\![43]\!] - 2[\![2]\!])$
v_5	0
v_6	$\dfrac{1}{2}([\![43]\!] - 2[\![2]\!])$

5_1:
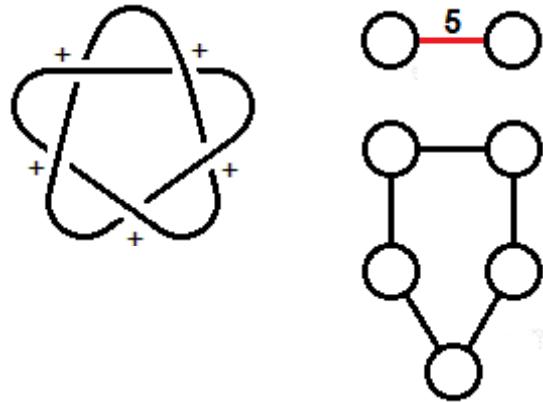
$v_{even} = x_{5.1}$

$v_{odd} = \pm y_{5.1}$

v_2	$3[\![2]\!]$
v_3	$\pm 5[\![3]\!]$
v_4	$[\![53]\!] + 3[\![52]\!] - 2[\![51]\!] + 3[\![2]\!]$
v_5	$\pm(2[\![54]\!] - [\![53]\!] + 5[\![3]\!])$
v_6	$[\![53]\!] + 3[\![52]\!] - 2[\![51]\!] + 3[\![2]\!]$

5_2:

$v_{even} = x_{5.2}$

$v_{odd} = \pm y_{5.2}$
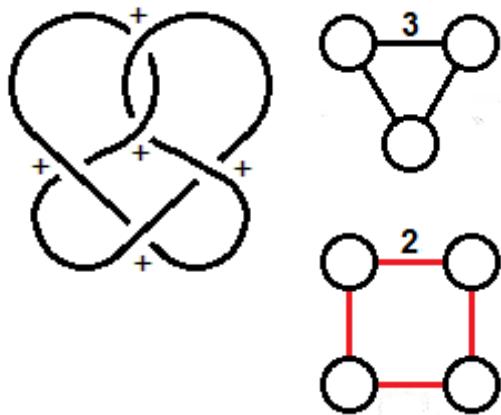
v_2	$2\llbracket 2 \rrbracket$
v_3	$\pm 3\llbracket 3 \rrbracket$
v_4	$\dfrac{1}{2}\left(\llbracket 43 \rrbracket + 2\llbracket 42 \rrbracket - 2\llbracket 41 \rrbracket + 4\llbracket 2 \rrbracket\right)$
v_5	$\pm\left(\llbracket 54 \rrbracket - \llbracket 53 \rrbracket + 3\llbracket 3 \rrbracket\right)$
v_6	$\dfrac{1}{2}\left(\llbracket 43 \rrbracket + 2\llbracket 42 \rrbracket - 2\llbracket 41 \rrbracket + 4\llbracket 2 \rrbracket\right)$

6_1:

$$v_{even} = x_{6.1}$$

$$v_{odd} = \pm y_{6.1}$$
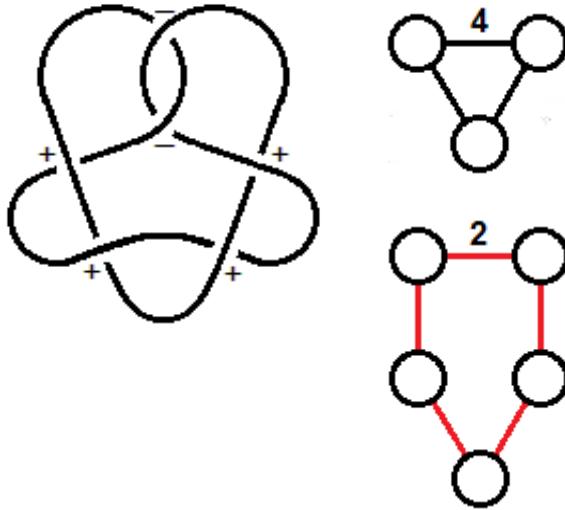
v_2	$-2[\![2]\!]$
v_3	$\mp[\![3]\!]$
v_4	$\dfrac{1}{2}\left(3[\![43]\!] - 2[\![42]\!] + 2[\![41]\!] - 4[\![2]\!]\right)$
v_5	$\pm\left([\![54]\!] - [\![53]\!] - [\![52]\!] + 2[\![51]\!] - [\![3]\!]\right)$
v_6	$\dfrac{1}{2}\left(2[\![69]\!] + 2[\![68]\!] - 4[\![67]\!] - [\![66]\!] - 5[\![65]\!] + 3[\![64]\!] + [\![63]\!] + [\![62]\!] \right.$ $\left. + 2[\![61]\!] + 3[\![43]\!] - 2[\![42]\!] + 2[\![41]\!] - 4[\![2]\!]\right)$

6_2:

$v_{even} = x_{6.2}$

$v_{odd} = \pm y_{6.2}$

v_2	$-[\![2]\!]$
v_3	$\mp[\![3]\!]$
v_4	$[\![43]\!] - 2[\![42]\!] + [\![41]\!] - [\![2]\!]$
v_5	$\pm([\![54]\!] - [\![53]\!] - 2[\![52]\!] + 3[\![51]\!] - [\![3]\!])$
v_6	$\frac{1}{2}(3[\![69]\!] + 5[\![68]\!] - 8[\![67]\!] + [\![66]\!] - 14[\![65]\!] + 5[\![64]\!] + 2[\![63]\!]$ $+ 2[\![62]\!] + 7[\![61]\!] + 2[\![43]\!] - 4[\![42]\!] + 2[\![41]\!] - 2[\![2]\!])$

6_3:

$v_{even} = x_{6.3}$

$v_{odd} = 0$

v_2	$[\![2]\!]$
v_3	0
v_4	$\dfrac{1}{2}(-[\![43]\!] + 2[\![42]\!] + 2[\![2]\!])$
v_5	0
v_6	$\dfrac{1}{2}(-[\![69]\!] - 3[\![68]\!] + 6[\![67]\!] - 5[\![66]\!] + 8[\![65]\!] - [\![64]\!]$ $+ [\![63]\!] - [\![62]\!] - 4[\![61]\!] - [\![43]\!] + 2[\![42]\!] + 2[\![2]\!])$

$6_{3.1\#3.1}$:

$$v_{even} = x_{3\#3}$$

$$v_{odd} = y_{3\#3}$$

v_2	$2[\![2]\!]$
v_3	$\pm 2[\![3]\!]$
v_4	$[\![41]\!] + 2[\![2]\!]$
v_5	$\pm(2[\![51]\!] - 2[\![3]\!])$
v_6	$\frac{1}{2}(2[\![69]\!] + 3[\![68]\!] - 12[\![67]\!] + 5[\![66]\!] - 2[\![65]\!]$ $+ 2[\![63]\!] - 2[\![62]\!] + [\![61]\!] + 2[\![41]\!] + 4[\![2]\!])$

$6_{3.1\#\overline{3.1}}$:

$v_{even} = x_{3\#\overline{3}}$

$v_{odd} = 0$

v_2	$2[\![2]\!]$
v_3	0
v_4	$[\![41]\!] + 2[\![2]\!]$
v_5	0
v_6	$\dfrac{1}{2}(2[\![69]\!] + 3[\![68]\!] - 12[\![67]\!] + 5[\![66]\!] - 2[\![65]\!] - 4[\![62]\!]$ $+ 2[\![63]\!] - 2[\![62]\!] + [\![61]\!] + 2[\![41]\!] + 4[\![2]\!])$

7_1:
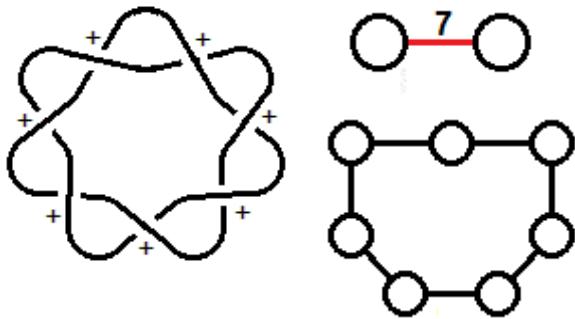
$$v_{even} = x_{7.1}$$

$$v_{odd} = \pm y_{7.1}$$

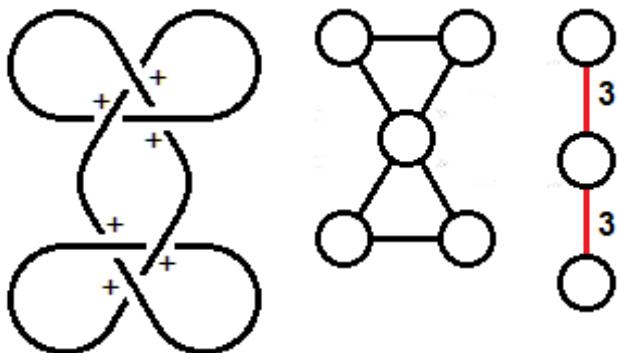| v_2	$6[\![2]\!]$
v_3	$\pm 14[\![3]\!]$
v_4	$5[\![43]\!] + 15[\![42]\!] - 10[\![41]\!] + 6[\![2]\!]$
v_5	$\pm(14[\![54]\!] - 7[\![53]\!] + 14[\![3]\!])$
v_6	$10[\![69]\!] + 18[\![68]\!] - 38[\![67]\!] + 13[\![66]\!] - 36[\![65]\!] + 6[\![64]\!] + 4[\![63]\!]$ $+ [\![62]\!] + 18[\![61]\!] + 5[\![43]\!] + 15[\![42]\!] - 10[\![41]\!] + 6[\![2]\!]$

7_2:

$v_{even} = x_{7.2}$

$v_{odd} = \pm y_{7.2}$
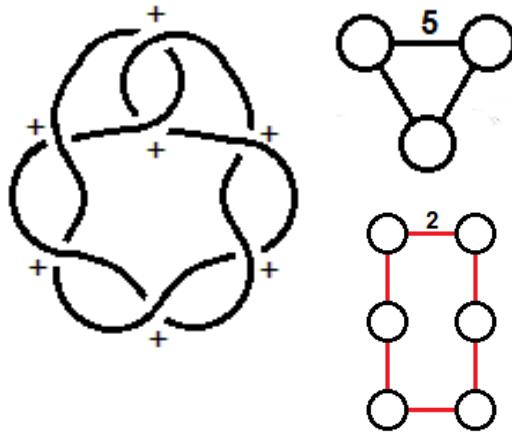
v_2	$3[\![2]\!]$
v_3	$\pm 6[\![3]\!]$
v_4	$\dfrac{1}{2}(3[\![43]\!] + 8[\![42]\!] - 8[\![41]\!] + 6[\![2]\!])$
v_5	$\pm(4[\![54]\!] - 4[\![53]\!] + [\![52]\!] - 2[\![51]\!] + 6[\![3]\!])$
v_6	$\dfrac{1}{2}(2[\![69]\!] + 2[\![68]\!] - 4[\![67]\!] - [\![66]\!] - 3[\![65]\!] + [\![64]\!] + [\![63]\!] - [\![62]\!]$ $+ 2[\![61]\!] + 3[\![43]\!] + 8[\![42]\!] - 8[\![41]\!] + 6[\![2]\!])$

7_3:
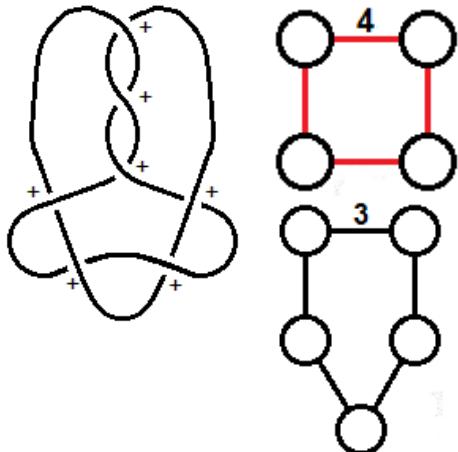
$$v_{even} = x_{7.3}$$
$$v_{odd} = \pm y_{7.3}$$

v_2	$5[\![2]\!]$
v_3	$\pm 11[\![3]\!]$
v_4	$4[\![43]\!] + 9[\![42]\!] - 7[\![41]\!] + 5[\![2]\!]$
v_5	$\pm(11[\![54]\!] - 8[\![53]\!] - 2[\![52]\!] + 3[\![51]\!] + 11[\![3]\!])$
v_6	$\frac{1}{2}(17[\![69]\!] + 31[\![68]\!] - 76[\![67]\!] + 29[\![66]\!] - 58[\![65]\!] + 7[\![64]\!] + 8[\![63]\!] - 2[\![62]\!] + 29[\![61]\!] + 8[\![43]\!] + 18[\![42]\!] - 14[\![41]\!] + 10[\![2]\!])$

7_4:
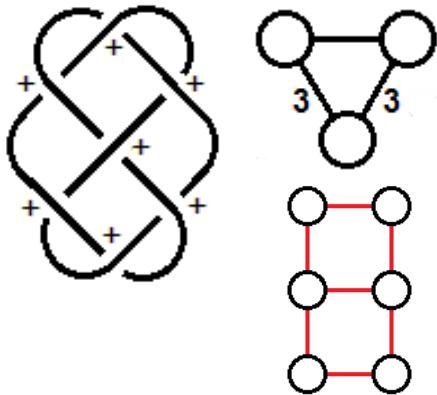
$$v_{even} = x_{7.4}$$

$$v_{odd} = \pm y_{7.4}$$

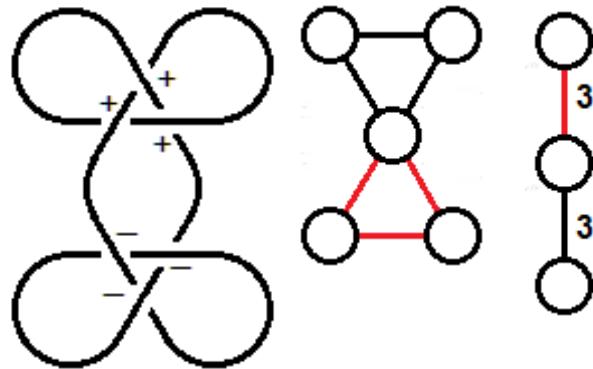| v_2	$4\llbracket 2 \rrbracket$
v_3	$\pm 8\llbracket 3 \rrbracket$
v_4	$3\llbracket 43 \rrbracket + 4\llbracket 42 \rrbracket - 4\llbracket 41 \rrbracket + 4\llbracket 2 \rrbracket$
v_5	$\pm(8\llbracket 54 \rrbracket - 8\llbracket 53 \rrbracket - 2\llbracket 52 \rrbracket + 4\llbracket 51 \rrbracket + 8\llbracket 3 \rrbracket)$
v_6	$7\llbracket 69 \rrbracket + 13\llbracket 68 \rrbracket - 36\llbracket 67 \rrbracket + 16\llbracket 66 \rrbracket - 22\llbracket 65 \rrbracket + 3\llbracket 63 \rrbracket - 2\llbracket 62 \rrbracket + 11\llbracket 61 \rrbracket + 3\llbracket 43 \rrbracket + 4\llbracket 42 \rrbracket - 4\llbracket 41 \rrbracket + 4\llbracket 2 \rrbracket$

7_5:

$v_{even} = x_{7.5}$

$v_{odd} = \pm y_{7.5}$
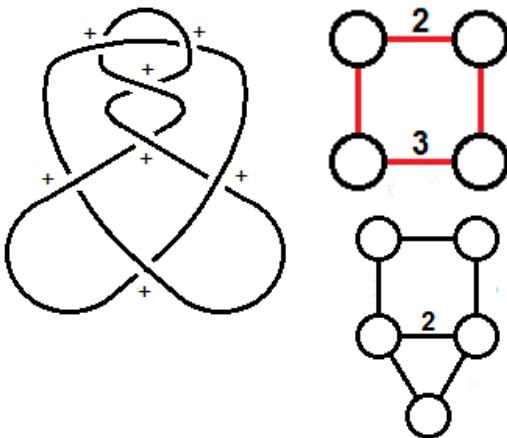
v_2	$4[\![2]\!]$
v_3	$\pm 8[\![3]\!]$
v_4	$2[\![43]\!] + 7[\![42]\!] - 5[\![41]\!] + 4[\![2]\!]$
v_5	$\pm(5[\![54]\!] - 3[\![53]\!] + 2[\![52]\!] - 3[\![51]\!] + 8[\![3]\!])$
v_6	$\dfrac{1}{2}(2[\![69]\!] + 2[\![68]\!] - 2[\![67]\!] - 2[\![66]\!] - 2[\![65]\!] + [\![63]\!] - [\![62]\!]$ $+ [\![61]\!] + 4[\![43]\!] + 14[\![42]\!] - 10[\![41]\!] + 8[\![2]\!])$

7_6:

$v_{even} = x_{7.6}$

$v_{odd} = \pm y_{7.6}$

v_2	$[\![2]\!]$
v_3	$\pm 2[\![3]\!]$
v_4	$\dfrac{1}{2}([\![43]\!] - 2[\![41]\!] + 2[\![2]\!])$
v_5	$\pm([\![54]\!] - 2[\![53]\!] + [\![52]\!] - [\![51]\!] + 2[\![3]\!])$
v_6	$\dfrac{1}{2}(2[\![67]\!] - [\![66]\!] - [\![65]\!] + [\![64]\!] + [\![61]\!] + [\![43]\!] - 2[\![41]\!] + 2[\![2]\!])$

7_7:
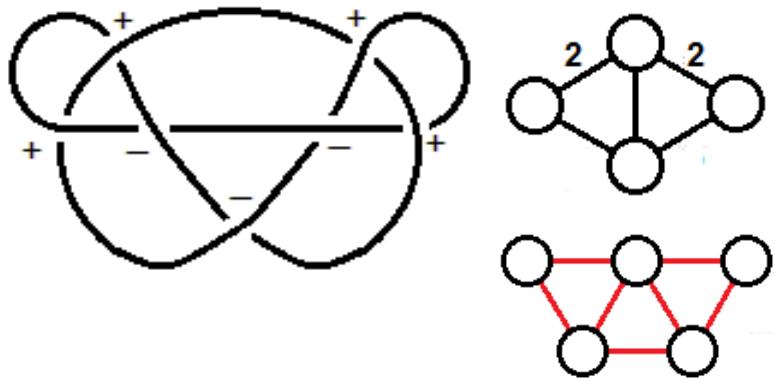
$v_{even} = x_{7.7}$

$v_{odd} = \pm y_{7.7}$

v_2	$-[\![2]\!]$
v_3	$\mp[\![3]\!]$
v_4	$[\![42]\!] - [\![2]\!]$
v_5	$\pm([\![53]\!] - 2[\![52]\!] + 2[\![51]\!] - [\![3]\!])$
v_6	$-2[\![67]\!] + [\![66]\!] + 2[\![65]\!] - [\![64]\!] - [\![62]\!]$ $- [\![61]\!] + [\![42]\!] - [\![2]\!]$

$7_{3.1\#4.1}$:
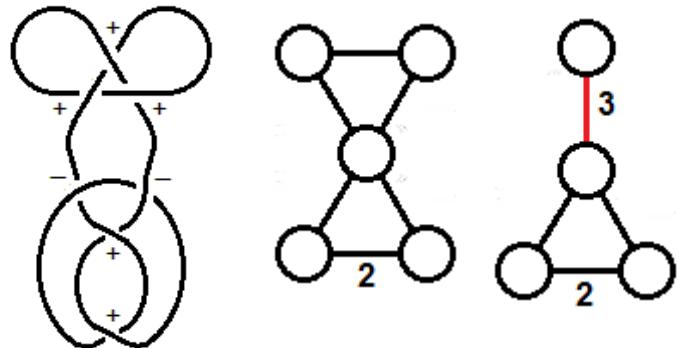
$$v_{even} = x_{3\#4}$$
$$v_{odd} = \pm y_{3\#4}$$

v_2	0
v_3	$\pm[\![3]\!]$
v_4	$\dfrac{1}{2}([\![43]\!] - 2[\![41]\!])$
v_5	$\mp([\![51]\!] - [\![3]\!])$
v_6	$\dfrac{1}{2}(-2[\![69]\!] - 3[\![68]\!] + 12[\![67]\!] - 5[\![66]\!] + 2[\![65]\!]$ $+ 2[\![64]\!] - [\![63]\!] + [\![62]\!] + [\![43]\!] - 2[\![41]\!])$

8_1:
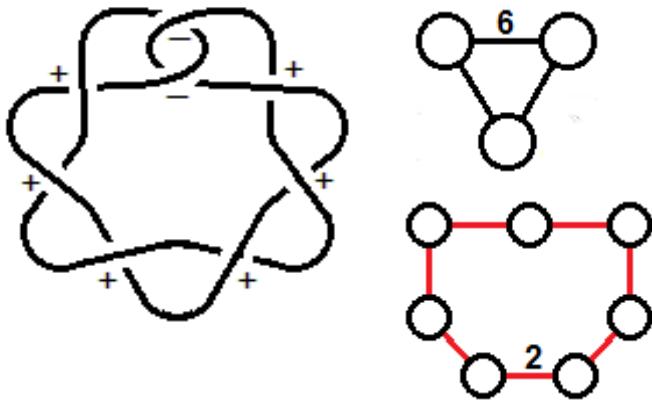
$v_{even} = x_{8.1}$

$v_{odd} = \pm y_{8.1}$

v_2	$-3[\![2]\!]$
v_3	$\mp 3[\![3]\!]$
v_4	$3[\![43]\!] - 4[\![42]\!] + 4[\![41]\!] - 3[\![2]\!]$
v_5	$\pm(4[\![54]\!] - 4[\![53]\!] - 5[\![52]\!] + 10[\![51]\!] - 3[\![3]\!])$
v_6	$\frac{1}{2}(10[\![69]\!] + 10[\![68]\!] - 20[\![67]\!] - 5[\![66]\!] - 27[\![65]\!] + 17[\![64]\!] + 5[\![63]\!]$ $+ 7[\![62]\!] + 10[\![61]\!] + 6[\![43]\!] - 8[\![42]\!] + 8[\![41]\!] - 6[\![2]\!])$

8_2:

$v_{even} = x_{8.2}$

$v_{odd} = \pm y_{8.2}$
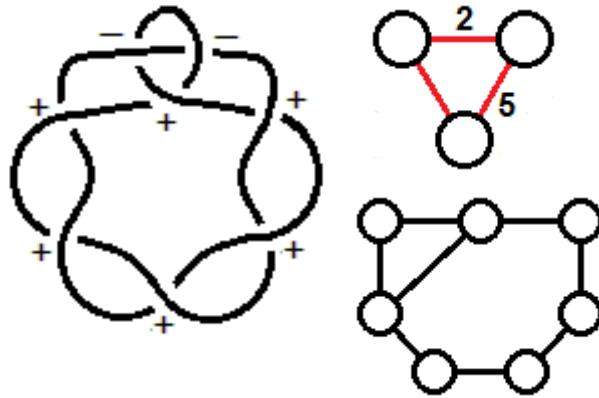
v_2	0
v_3	$\mp[\![3]\!]$
v_4	$\dfrac{1}{2}(3[\![43]\!] - 12[\![42]\!] + 6[\![41]\!])$
v_5	$\pm(3[\![54]\!] - 4[\![53]\!] - 8[\![52]\!] + 12[\![51]\!] - [\![3]\!])$
v_6	$\dfrac{1}{2}(12[\![69]\!] + 20[\![68]\!] - 40[\![67]\!] + 8[\![66]\!] - 56[\![65]\!] + 20[\![64]\!] + 10[\![63]\!] + 6[\![62]\!] + 28[\![61]\!] + 3[\![43]\!] - 12[\![42]\!] + 6[\![41]\!])$

8_3:

$$v_{even} = x_{8.3}$$

$$v_{odd} = 0$$

v_2	$-4[\![2]\!]$
v_3	0
v_4	$5[\![43]\!] - 4[\![42]\!] + 4[\![41]\!] - 4[\![2]\!]$
v_5	0
v_6	$[\![69]\!] - 5[\![68]\!] + 20[\![67]\!] - 20[\![66]\!] + 2[\![65]\!] + 12[\![64]\!] + [\![63]\!]$ $+ 6[\![62]\!] - 3[\![61]\!] + 5[\![43]\!] - 4[\![42]\!] + 4[\![41]\!] - 4[\![2]\!]$

8_4:

$$v_{even} = x_{8.4}$$

$$v_{odd} = \pm y_{8.4}$$

v_2	$-3[\![2]\!]$
v_3	$\pm[\![3]\!]$
v_4	$4[\![43]\!] - 5[\![42]\!] + 3[\![41]\!] - 3[\![2]\!]$
v_5	$\mp([\![54]\!] - 4[\![52]\!] + 5[\![51]\!] - [\![3]\!])$
v_6	$\frac{1}{2}(3[\![69]\!] - 3[\![68]\!] + 28[\![67]\!] - 29[\![66]\!] - 16[\![65]\!] + 23[\![64]\!] + 4[\![63]\!]$ $+ 12[\![62]\!] + 7[\![61]\!] + 8[\![43]\!] - 10[\![v42]\!] + 6[\![41]\!] - 6[\![2]\!])$

8_5:
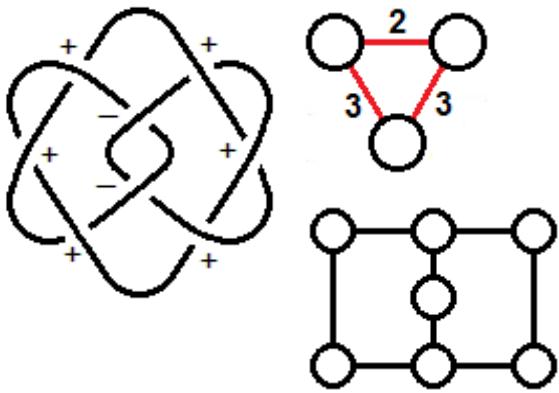
$$v_{even} = x_{8.5}$$

$$v_{odd} = \pm y_{8.5}$$

v_2	$-[\![2]\!]$
v_3	$\mp 3[\![3]\!]$
v_4	$2[\![43]\!] - 8[\![42]\!] + 5[\![41]\!] - [\![2]\!]$
v_5	$\pm(4[\![54]\!] - 4[\![53]\!] - 12[\![52]\!] + 18[\![51]\!] - 3[\![3]\!])$
v_6	$\frac{1}{2}(16[\![69]\!] + 25[\![68]\!] - 44[\![67]\!] + [\![66]\!] - 74[\![65]\!] + 34[\![64]\!] + 14[\![63]\!]$ $+ 10[\![62]\!] + 37[\![61]\!] + 4[\![43]\!] - 16[\![42]\!] + 10[\![41]\!] - 2[\![2]\!])$

8_6:

$$v_{even} = x_{8.6}$$

$$v_{odd} = \pm y_{8.6}$$
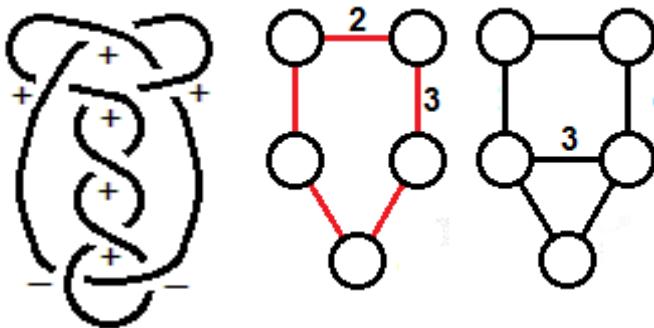
v_2	$-2[\![2]\!]$
v_3	$\mp 3[\![3]\!]$
v_4	$\dfrac{1}{2}(5[\![43]\!] - 12[\![42]\!] + 8[\![41]\!] - 4[\![2]\!])$
v_5	$\pm(4[\![54]\!] - 4[\![53]\!] - 8[\![52]\!] + 13[\![51]\!] - 3[\![3]\!])$
v_6	$\frac{1}{2}(12[\![69]\!] + 16[\![68]\!] - 26[\![67]\!] - 4[\![66]\!] - 48[\![65]\!] + 24[\![64]\!] + 9[\![63]\!]$ $+ 9[\![62]\!] + 23[\![61]\!] + 5[\![43]\!] - 12[\![42]\!] + 8[\![41]\!] - 4[\![2]\!])$

8_7:

$$v_{even} = x_{8.7}$$

$$v_{odd} = \pm y_{8.7}$$

v_2	$2[\![2]\!]$
v_3	$\pm 2[\![3]\!]$
v_4	$-[\![43]\!] + 4[\![42]\!] - [\![41]\!] + 2[\![2]\!]$
v_5	$\mp([\![54]\!] - 2[\![53]\!] - 2[\![52]\!] + 3[\![51]\!] - 2[\![3]\!])$
v_6	$\frac{1}{2}(-6[\![69]\!] - 14[\![68]\!] + 34[\![67]\!] - 22[\![66]\!] + 34[\![65]\!] - 4[\![64]\!] + [\![63]\!] \\ - [\![62]\!] - 17[\![61]\!] - 2[\![43]\!] + 8[\![42]\!] - 2[\![41]\!] + 4[\![2]\!])$

8_8:

$v_{even} = x_{8.8}$

$v_{odd} = \pm y_{8.8}$

v_2	$2[\![2]\!]$
v_3	$\pm[\![3]\!]$
v_4	$\dfrac{1}{2}(-[\![43]\!] + 4[\![42]\!] + 4[\![2]\!])$
v_5	$\pm([\![52]\!] - [\![51]\!] + [\![3]\!])$
v_6	$\dfrac{1}{2}(-2[\![69]\!] - 6[\![68]\!] + 14[\![67]\!] - 13[\![66]\!] + 17[\![65]\!] - 3[\![64]\!] + 4[\![63]\!]$ $- 2[\![62]\!] - 9[\![61]\!] - [\![43]\!] + 4[\![42]\!] + 4[\![2]\!])$

8_9:

$$v_{even} = x_{8.9}$$
$$v_{odd} = 0$$

v_2	$-2[\![2]\!]$
v_3	0
v_4	$3[\![43]\!] - 5[\![42]\!] + 2[\![41]\!] - 2[\![2]\!]$
v_5	0
v_6	$\frac{1}{2}(3[\![69]\!] + [\![68]\!] + 18[\![67]\!] - 19[\![66]\!] - 24[\![65]\!] + 19[\![64]\!] + 5[\![63]\!] + 9[\![62]\!] + 12[\![61]\!] + 6[\![43]\!] - 10[\![42]\!] + 4[\![41]\!] - 4[\![2]\!])$

8_{10}:
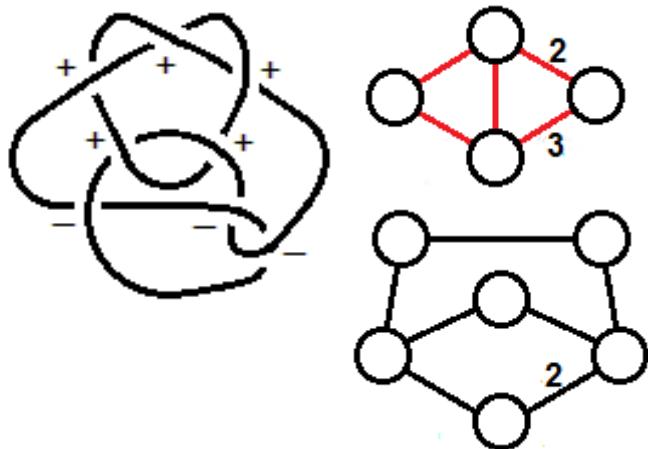
$$v_{even} = x_{8.10}$$

$$v_{odd} = \pm y_{8.10}$$

v_2	$3[\![2]\!]$
v_3	$\pm 3[\![3]\!]$
v_4	$3[\![42]\!] + 3[\![2]\!]$
v_5	$\pm([\![54]\!] - [\![53]\!] + 2[\![52]\!] - [\![51]\!] + 3[\![3]\!])$
v_6	$\dfrac{1}{2}(-3[\![68]\!] + 2[\![67]\!] - 7[\![66]\!] + 16[\![65]\!] - 8[\![64]\!]$ $+ 5[\![63]\!] - 3[\![62]\!] - 8[\![61]\!] + 6[\![42]\!] + 6[\![2]\!])$

8_{11}:

$$v_{even} = x_{8.11}$$
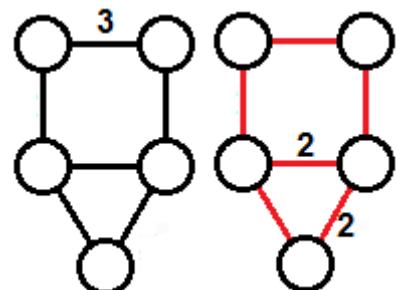
$$v_{odd} = y_{8.11}$$

v_2	$-[\![2]\!]$
v_3	$\mp 2[\![3]\!]$
v_4	$\dfrac{1}{2}(3[\![43]\!] - 10[\![42]\!] + 6[\![41]\!] - 2[\![2]\!])$
v_5	$\pm(3[\![54]\!] - 4[\![53]\!] - 5[\![52]\!] + 9[\![51]\!] - 2[\![3]\!])$
v_6	$\frac{1}{2}(11[\![69]\!] + 17[\![68]\!] - 36[\![67]\!] + 8[\![66]\!] - 43[\![65]\!] + 14[\![64]\!] + 7[\![63]\!] + 5[\![62]\!] + 21[\![61]\!] + 3[\![43]\!] - 10[\![42]\!] + 6[\![41]\!] - 2[\![2]\!])$

8_{12}:

$v_{even} = x_{8.12}$

$v_{odd} = 0$
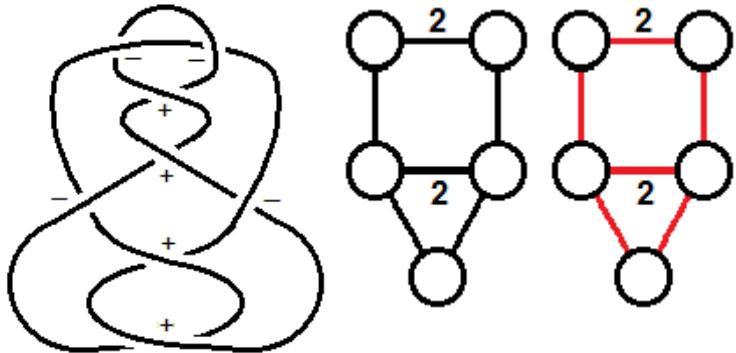
v_2	$-3[\![2]\!]$
v_3	0
v_4	$\dfrac{1}{2}(5[\![43]\!] - 2[\![42]\!] + 4[\![41]\!] - 6[\![2]\!])$
v_5	0
v_6	$\dfrac{1}{2}(3[\![69]\!] + [\![68]\!] - 6[\![67]\!] - [\![66]\!] - 2[\![65]\!] + 3[\![64]\!] - [\![63]\!]$ $+ 3[\![62]\!] - 2[\![61]\!] + 5[\![43]\!] - 2[\![42]\!] + 4[\![41]\!] - 6[\![2]\!])$

8_{13}:

$v_{even} = x_{8.13}$

$v_{odd} = \pm y_{8.13}$

v_2	$[\![2]\!]$
v_3	$\pm[\![3]\!]$
v_4	$-[\![43]\!] + 3[\![42]\!] - [\![41]\!] + [\![2]\!]$
v_5	$\mp([\![54]\!] - 2[\![53]\!] + [\![51]\!] - [\![3]\!])$
v_6	$\frac{1}{2}(-5[\![69]\!] - 11[\![68]\!] + 28[\![67]\!] - 19[\![66]\!] + 26[\![65]\!] - [\![64]\!] + 2[\![63]\!]$ $- 2[\![62]\!] - 13[\![61]\!] - 2[\![43]\!] + 6[\![42]\!] - 2[\![41]\!] + 2[\![2]\!])$

8_{14}:

$$v_{even} = x_{8.14}$$

$$v_{odd} = \pm y_{8.14}$$

v_2	0
v_3	0
v_4	$[\![43]\!] - 3[\![42]\!] + [\![41]\!]$
v_5	$\pm([\![54]\!] - [\![53]\!] - 4[\![52]\!] + 5[\![51]\!])$
v_6	$\frac{1}{2}(4[\![69]\!] + 8[\![68]\!] - 14[\![67]\!] + 6[\![66]\!] - 26[\![65]\!] + 8[\![64]\!] + [\![63]\!] + 5[\![62]\!]$ $+ 13[\![61]\!] + 2[\![43]\!] - 6[\![42]\!] + 2[\![41]\!])$

8_{15}:
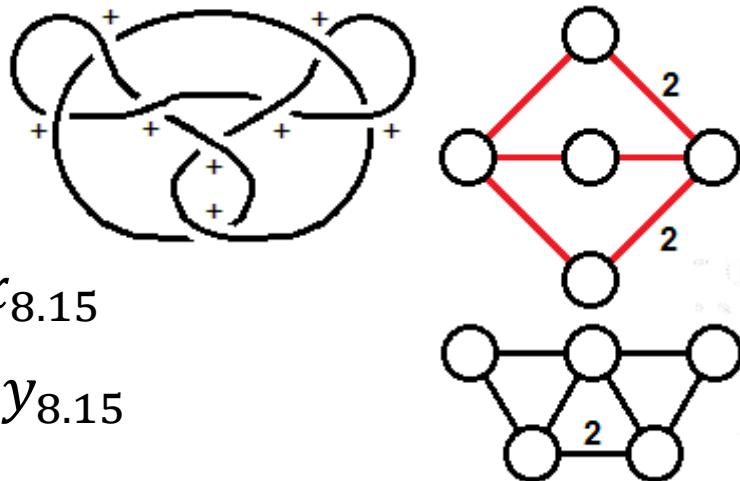
$$v_{even} = x_{8.15}$$

$$v_{odd} = \pm y_{8.15}$$

v_2	$4[\![2]\!]$
v_3	$\pm 7[\![3]\!]$
v_4	$\dfrac{1}{2}(3[\![43]\!] + 10[\![42]\!] - 4[\![41]\!] + 8[\![2]\!])$
v_5	$\pm(3[\![54]\!] - [\![53]\!] + [\![52]\!] + 2[\![51]\!] + 7[\![3]\!])$
v_6	$\dfrac{1}{2}(2[\![69]\!] + 3[\![68]\!] - 12[\![67]\!] + 8[\![66]\!] - 3[\![65]\!] + [\![64]\!] + [\![63]\!]$ $- [\![62]\!] + [\![61]\!] + 3[\![43]\!] + 10[\![42]\!] - 4[\![41]\!] + 8[\![2]\!])$

8_{16}:

$$v_{even} = x_{8.16}$$

$$v_{odd} = \pm y_{8.16}$$

v_2	$[\![2]\!]$
v_3	$\pm[\![3]\!]$
v_4	$-[\![43]\!] + 3[\![42]\!] - [\![41]\!] + [\![2]\!]$
v_5	$\mp([\![53]\!] - 4[\![52]\!] + 4[\![51]\!] - [\![3]\!])$
v_6	$\frac{1}{2}(-4[\![69]\!] - 9[\![68]\!] + 20[\![67]\!] - 15[\![66]\!] + 30[\![65]\!] - 6[\![64]\!] + 2[\![63]\!]$ $- 4[\![62]\!] - 15[\![61]\!] - 2[\![43]\!] + 6[\![42]\!] - 2[\![41]\!] + 2[\![2]\!])$

8_{17}:

$$v_{even} = x_{8.17}$$

$$v_{odd} = 0$$
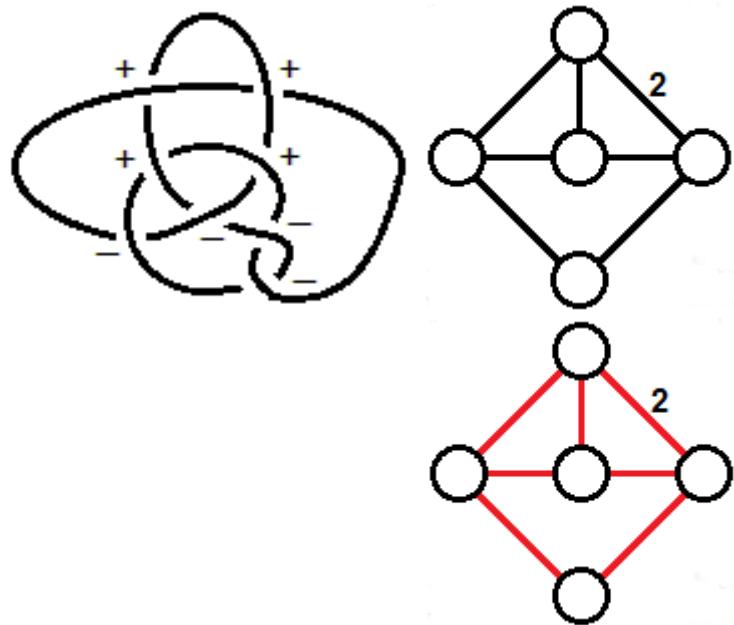
v_2	$-[\![2]\!]$
v_3	0
v_4	$\dfrac{1}{2}(3[\![43]\!] - 6[\![42]\!] + 2[\![41]\!] - 2[\![2]\!])$
v_5	0
v_6	$\dfrac{1}{2}(3[\![69]\!] + 4[\![68]\!] - 2[\![67]\!] - 22[\![65]\!] + 9[\![64]\!] + [\![63]\!] + 5[\![62]\!] + 11[\![61]\!] + 3[\![43]\!] - 6[\![42]\!] + 2[\![41]\!] - 2[\![2]\!])$

8_{18}:

$v_{even} = x_{8.18}$

$v_{odd} = 0$

v_2	$[\![2]\!]$
v_3	0
v_4	$\dfrac{1}{2}([\![43]\!] - 4[\![42]\!] + 2[\![41]\!] + 2[\![2]\!])$
v_5	0
v_6	$\dfrac{1}{2}(4[\![69]\!] + 7[\![68]\!] - 20[\![67]\!] + 15[\![66]\!] - 22[\![65]\!] + 2[\![64]\!] - 2[\![63]\!] + 2[\![62]\!] + 11[\![61]\!] + [\![43]\!] - 4[\![42]\!] + 2[\![41]\!] + 2[\![2]\!])$

8_{19}:

$$v_{even} = x_{8.19}$$

$$v_{odd} = \pm y_{8.19}$$
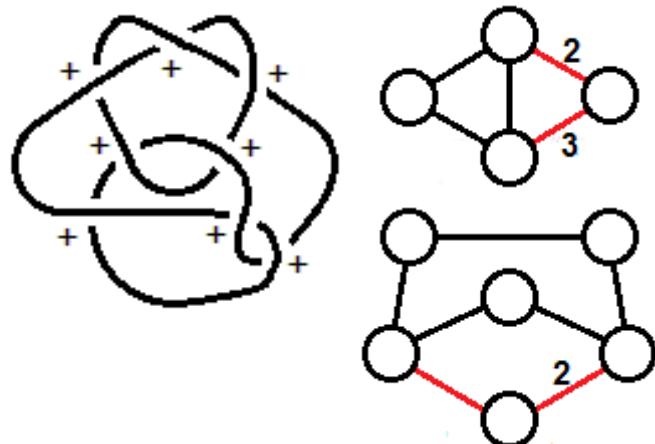
v_2	$5[\![2]\!]$
v_3	$\pm 10[\![3]\!]$
v_4	$\dfrac{1}{2}(5[\![43]\!] + 20[\![42]\!] - 10[\![41]\!] + 10[\![2]\!])$
v_5	$\pm(6[\![54]\!] - 2[\![53]\!] + 4[\![52]\!] - 2[\![51]\!] + 10[\![3]\!])$
v_6	$\dfrac{1}{2}(4[\![69]\!] + 5[\![68]\!] - 12[\![67]\!] + 5[\![66]\!] - 2[\![65]\!] - 2[\![64]\!] + 2[\![63]\!] - 2[\![62]\!]$ $+ [\![61]\!] + 5[\![43]\!] + 20[\![42]\!] - 10[\![41]\!] + 10[\![2]\!])$

8_{20}:
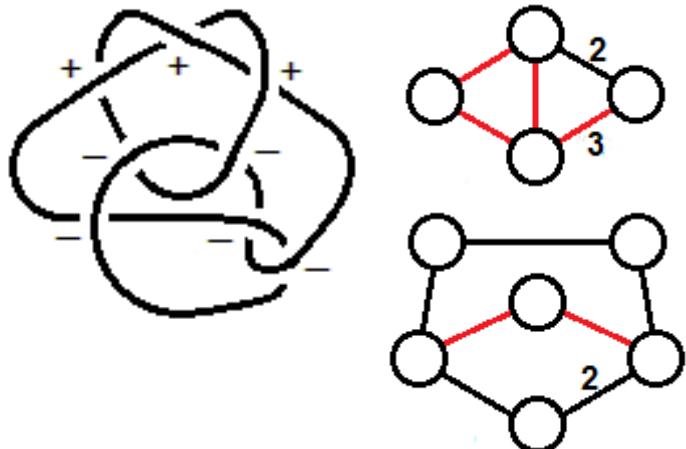
$v_{even} = x_{8.20}$

$v_{odd} = \pm y_{8.20}$

v_2	$2[\![2]\!]$
v_3	$\mp 2[\![3]\!]$
v_4	$[\![42]\!] + 2[\![2]\!]$
v_5	$\mp([\![54]\!] - 2[\![53]\!] + 2[\![52]\!] - [\![51]\!] + 2[\![3]\!])$
v_6	$\dfrac{1}{2}([\![69]\!] - 4[\![67]\!] + 4[\![65]\!] - 3[\![64]\!] + 2[\![63]\!]$ $- 2[\![62]\!] - 2[\![61]\!] + 2[\![42]\!] + 4[\![2]\!])$

8_{21}:

$$v_{even} = x_{8.21}$$

$$v_{odd} = \pm y_{8.21}$$

v_2	0
v_3	$\pm[\![3]\!]$
v_4	$\dfrac{1}{2}([\![43]\!] - 6[\![42]\!] + 4[\![41]\!])$
v_5	$\mp([\![54]\!] - [\![53]\!] - 4[\![52]\!] + 6[\![51]\!] - [\![3]\!])$
v_6	$\dfrac{1}{2}(6[\![69]\!] + 11[\![68]\!] - 24[\![67]\!] + 7[\![66]\!] - 26[\![65]\!] + 8[\![64]\!] + 4[\![63]\!] + 2[\![62]\!] + 13[\![61]\!] + [\![43]\!] - 6[\![42]\!] + 4[\![41]\!])$